Limitations of the Near-Wall k-ϵ Turbulence Model

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Comparisons are given at two different Reynolds numbers between the measured turbulent kinetic energy in channel flow and the predictions of several near-wall variants of the k-ϵ closure. Included among these are new calculations based on a form of the ϵ equation that is consistent with the physically correct boundary condition. It is observed that all of the approaches fail to account for the large peak k value in the wall region that is evident in the experimental data. By contrasting the computed energy budget with its measured values it is shown that this defect may be attributed to a fundamental inconsistency in the commonly used model for the pressure diffusion term in the k equation near the boundary.

Introduction

THE k-ϵ closure is widely utilized today in the prediction of the mean properties of turbulent flowfields. In recent years, extensions of its basic form have been developed to permit treatment of complex fluid motions, such as those found in curved pipes,[1] compressible boundary layers,[2] combustion chambers,[3] etc. A strong incentive for the continued development and application of the k-ϵ model has been the remarkable success it has achieved in the prediction of mean velocity fields in wall-bounded shear flows, such as occur in channels, pipes, and boundary layers.[4-6] The success of the k-ϵ closure in these relatively simple flows is not complete, however, since the high accuracy obtained for mean velocity is not duplicated in the case of the turbulent kinetic energy, k. A notable defect[4,7-9] is the 25% or more underprediction of the large peak k value shown by Clark[10] and Kreplin and Eckelmann[11] to occur at $y^+ \approx 15$ in a channel flow. For applications where predictions of turbulence levels are important, such as in internal combustion engine simulations,[12] this failure can be of great practical significance.

It may be expected that major errors in the prediction of k stem from deficiencies in the low turbulent Reynolds number forms of the k-ϵ equations which have been developed for the region adjacent to boundaries. Jones and Launder[4] formulated the first such model that permitted k and ϵ to be calculated down to a solid wall without the use of wall functions. Several more recent studies, including those of Lam and Bremhorst,[7] Hassid and Poreh,[8] and Chien,[9] have attempted to improve upon the Jones and Launder approach, but these authors report only modest gains in the accuracy of k.

The object of the present study is to elucidate the cause of this discrepancy by making direct comparisons between the terms in the exact and modeled energy equations. To accomplish this, a set of new calculations was performed in which ϵ was required to satisfy the physically correct boundary condition at the wall. It was revealed that the failure of the k-ϵ model to predict the correct peak k value is most likely due to reliance on a flawed model for the pressure-velocity correlation in the k equation. This relation will be shown to be inconsistent with the experimentally measured values of the exact term as found by Kreplin and Eckelmann,[11] and may be directly implicated in suppressing the peak k value. Removal of this limitation should allow for better predictions of k near solid boundaries.

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Turbulence Equations

In the case of fully developed turbulent channel flow, the exact k equation is given by

$$0 = \underset{\text{GD}}{\nu k,_{22}} - \underset{\text{PD}}{\frac{1}{\rho}\overline{pu_{2,2}}} - \underset{\text{KD}}{\frac{1}{2}\overline{u_2 u_i^2}_{,2}} - \underset{\text{P}}{\overline{u_1 u_2} U_{1,2}} - \underset{\text{D}}{\epsilon} = 0 \tag{1}$$

where GD denotes the gradient diffusion term, PD the pressure diffusion term, KD accounts for the kinetic energy diffusion, P represents production, and D dissipation. Also, x_1, x_2, and x_3 are the streamwise, normal, and spanwise coordinates, respectively, $(\ ),_i \equiv \partial(\ )/\partial x_i$, u_i is the velocity fluctuation vector, p the pressure fluctuation, and U_1 the mean streamwise velocity. The widely accepted closed form of the k equation consists of

$$\underset{\text{GD}}{\nu k,_{22}} + \underset{\text{PD+KD}}{(\nu_T k,_2),_2} + \underset{\text{P}}{\nu_T U_{1,2}^2} - \underset{\text{D}}{\epsilon} = 0 \tag{2}$$

where ν_T is the eddy viscosity. In the current study, numerical solutions for the terms in Eq. (2) will be compared to experimental evaluations of the equivalent terms in Eq. (1). Consequently, it is necessary that the isotropic dissipation rate, ϵ, appearing in Eq. (2) obeys the physically correct boundary condition[13,14]

$$\epsilon(0) = \phi(0) \tag{3}$$

where $\phi(y) \equiv 2\nu(\sqrt{k},_2)^2$. In contrast, most previous applications of the k-ϵ approach to wall-bounded flows use the artificial condition $\epsilon(0) = 0$. This creates the need to add an additional term in Eq. (2) to obtain the proper energy dissipation rate at the wall. For example, Jones and Launder[4] used $-\phi$, while Chien[9] incorporated the expression $-2\nu k/x_2^2$. In these cases, the term D in Eq. (2) is given by the sum of ϵ plus the additional expression.

For the present study a new set of calculations was performed in which Eq. (1) was solved together with the ϵ equation in the form

$$0 = C_1 \frac{\epsilon}{k}\text{P} - C_s \frac{\phi}{k}\text{P} - C_2 f_2 \frac{\epsilon(\epsilon - \phi)}{k} + C_s \nu_T \frac{\phi(\epsilon - \phi)}{k} + \left(\frac{\nu_T}{\sigma}\epsilon,_2\right),_2 + \nu\epsilon,_{22} + \frac{2\nu\nu_T}{\sigma}(U_{1,22})^2 \tag{4}$$

which has the virtue that ϵ satisfies Eq. (3) as a boundary condition. Here C_1, C_2, and C_s are constants, and f_2 is a

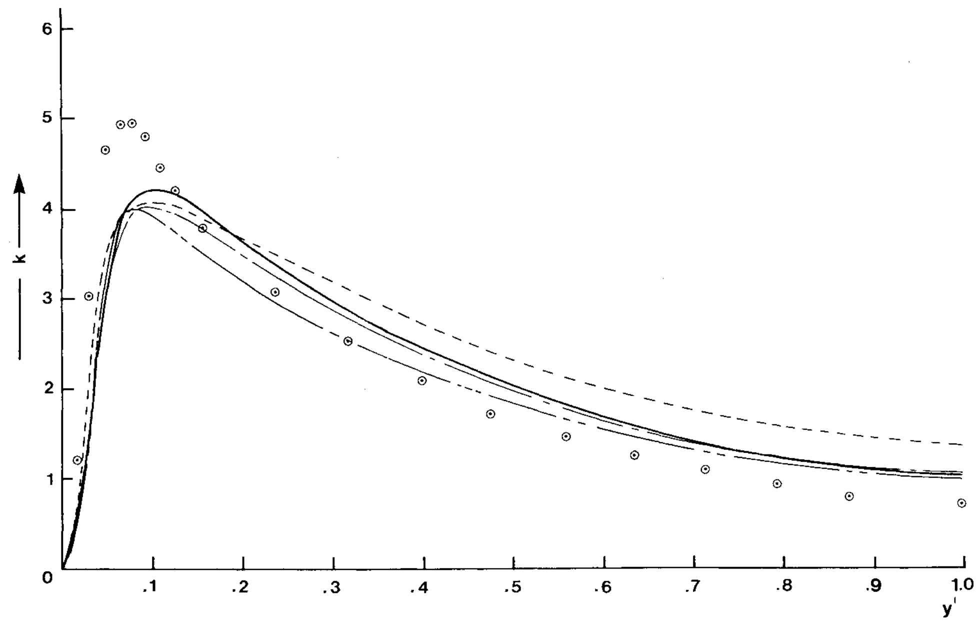

Fig. 1 Turbulent kinetic energy at $R = 3850$; —— present model; ----, Hassid and Poreh; -·-·-, Chien; -·-·-·-, Lam and Bremhorst; ⊙ data of Kreplin and Eckelmann.

function of $R_T \equiv k^2/\nu\epsilon$. With the exception of the second and fourth terms on the right-hand side, Eq. (4) is equivalent to the ϵ equation proposed by Hanjalic and Launder[13] in the context of their development of a Reynolds stress equation closure. The additional terms in Eq. (4) arise from an alternative derivation of the ϵ equation through simplification of the closed vorticity covariance equation.[15,16] It was found, however, that they only made a minor contribution to the calculated solution and could be dropped without significantly altering the results.

The eddy viscosity, ν_T, requires some special consideration in the vicinity of the boundary. Generally it may be assumed that $\nu_T = C_\mu f_\mu k^2/\epsilon_d$, where f_μ is a function of x_2 and $\epsilon_d = \epsilon - \phi$. In the present investigation $f_\mu = 1 - \exp(-0.0115x_2)$, which follows the approach of Chien.[9] This choice ensures that ν_T has the necessary x_2^3 dependence near the wall.

Numerical Solutions

Figure 1 shows the predicted values of k for the present and three other models at Reynolds number $R = 3850$, where R is based on the centerline velocity V_0 and channel halfwidth d, and $y = x_2/d$. The numerical solutions to Eqs. (2) and (4) given here were obtained using the algorithm described by Chien.[9] For the current model, $C_\mu = 0.09$, $C_1 = 1.5$, $C_2 = 2.0$, and $C_s = 0.45$. The constants were determined by holding C_μ and C_2 fixed and then adjusting C_1 and C_s until R_* and V_0/u_* agreed with the experimental values of 196.4 and 19.6, respectively. Here $R_* = u_* d/\nu$ and u_* is the friction velocity. It is seen that all of the different models significantly underpredict the experimental data of Kreplin and Eckelmann[11] near the wall, although the present approach represents a small improvement over the other models. The computed values of U_1/V_0 for all of the models correspond closely to the experimental data.

A comparison in the region $0 \le y^+ \le 70$ of the computed solution for k at $R = 15{,}200$ is shown in Fig. 2 for the same four models. It is seen that while the current model does offer some improvement in the prediction of k, once again all of the approaches fail to reproduce the large peak that is visible in the data of Ref. 10.

To determine the source of the discrepancies between theory and experiment evident in Figs. 1 and 2, the accuracy of the modeling inherent in Eq. (2) may be tested by comparing the computed energy budget to experimental measurements of the terms in Eq. (1). Such a comparison is made possible by a recent determination of the terms in the exact energy equation near walls[17] in which Taylor series expansions of the terms in Eq. (1) were evaluated for $0 \le y^+ \le 2$ using the experimental data of Eckelmann[18] and Kreplin and Eckelmann.[11] By combining these curves with those of earlier investigators such as Townsend,[19] for whom accurate data in the region $0 \le y^+ \le 7$ was not available, a revised budget may be constructed with substantially greater accuracy near the wall. For the region $2 \le y^+ \le 7$, the budget may be obtained by smoothly fairing in the curves for $y^+ \le 2$ and $y^+ \ge 7$. (These are denoted by dashed lines in the subsequent figures.) A comparison of the measured budget with the computed terms in Eq. (2) obtained from the solution at $R = 3850$ is given in Figs. 3-5. All quantities used here are nondimensionalized using u_* and ν.

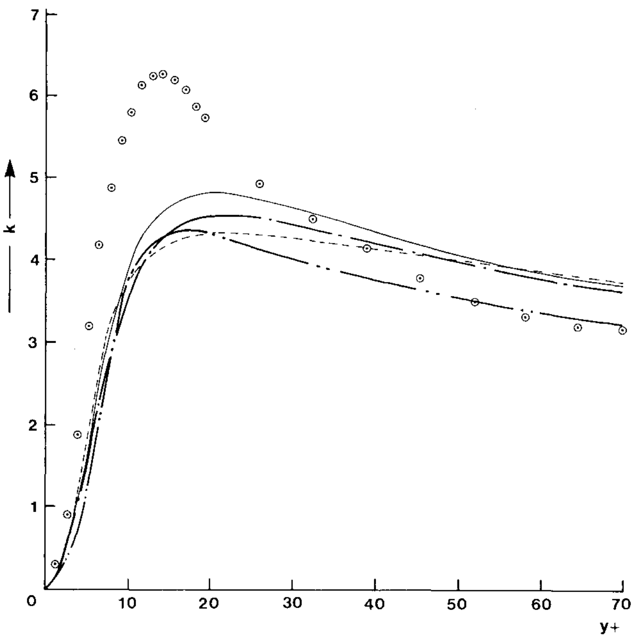

Fig. 1 Turbulence kinetic energy at $R = 15{,}200$;——, present model; ----, Hassid and Poreh; -·-·-, Chien; -·-·-·-, Lam and Bremhorst; ⊙ data of Clark.

Figure 3 contains the computed and experimental curves for the production and dissipation terms. It is evident that the computed production term agrees closely with experiment. This is to be expected as long as the predicted values of U_1 are accurate, since $P = -\overline{u_1 u_2} U_{1,2}$ and $\overline{u_1 u_2} = \nu U_{1,2} - u_*^2(1-y)$. In the case of the dissipation term the

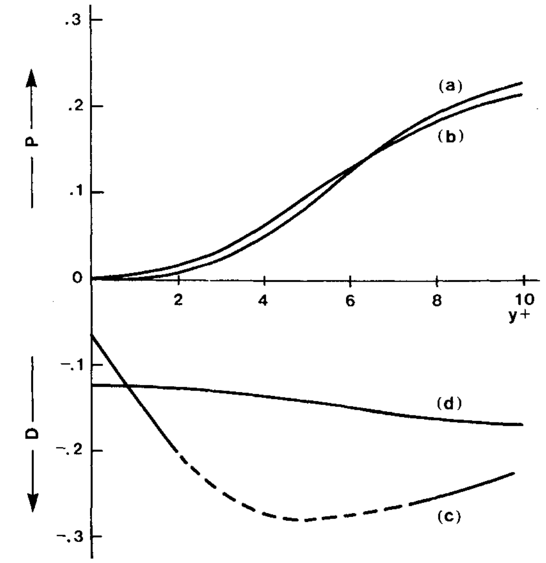

Fig. 3 Comparison of computed and experimental production and dissipation terms in energy equation. (a) measured production term; (b) computed values of term P in Eq. (1); (c) measured dissipation terms; (d) computed values of term D in Eq. (1).

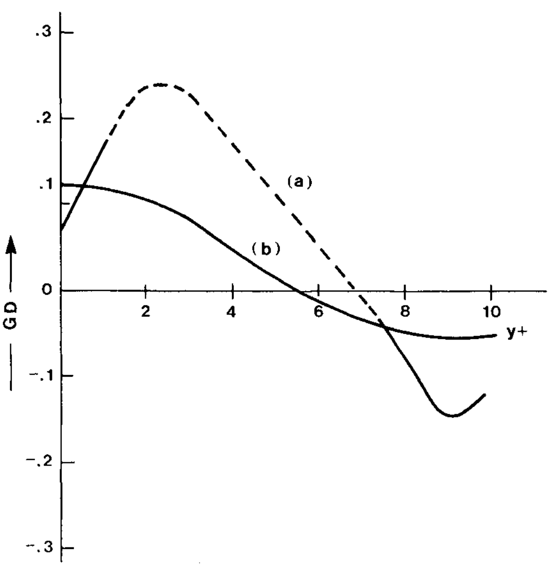

Fig. 4 Comparison of experimental, (a), and computed, (b), gradient diffusion terms in energy equation.

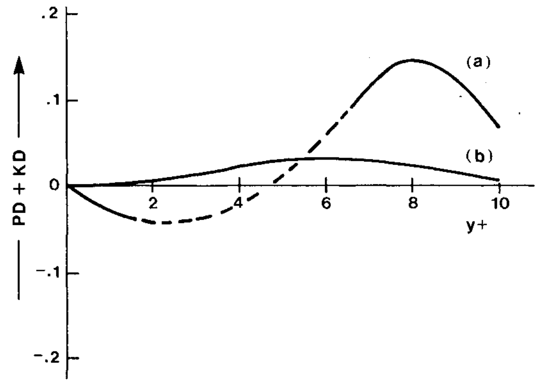

Fig. 5 Comparison of experimental, (a), and computed, (b), term PD + KD in energy equation.

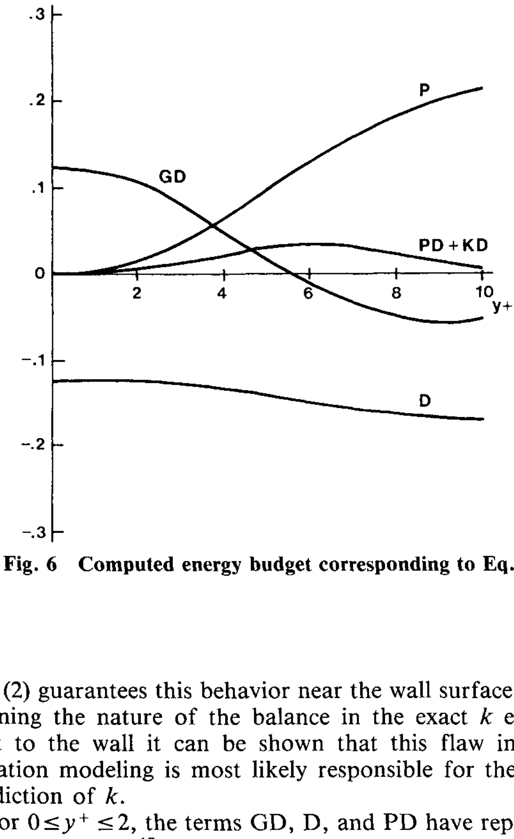

Fig. 6 Computed energy budget corresponding to Eq. (1).

agreement between the curves is not very good. This may be attributed to both the dependence of the ϵ equation on k, which is known to be inaccurate, and to possible shortcomings in the ϵ-equation models used in this study. However, it is clear from Figs. 1 and 2 that the predicted peak k value is largely insensitive to variations in the ϵ-equation model, which constitutes the most significant difference between the approaches.

The computed and experimental gradient diffusion terms are compared in Fig. 4. In this case, the obvious discrepancy may be attributed to the inaccuracy in the prediction of k. This is confirmed by the fact that the values of $k_{,22}$ calculated from numerical differentiation of the experimental data for k given in Fig. 1 are consistent with curve (a) in Fig. 4.

Figure 5 compares the computed and experimental curve for PD + KD. Here a major inconsistency is revealed to occur in that curve (a) representing the experiment has both a negative slope at the wall and is negative near it, while curve (b) representing the term PD + KD in Eq. (2) has zero slope and is everywhere positive. The form of the PD + KD term in Eq. (2) guarantees this behavior near the wall surface. By examining the nature of the balance in the exact k equation next to the wall it can be shown that this flaw in the k-equation modeling is most likely responsible for the underprediction of k.

For $0 \le y^+ \le 2$, the terms GD, D, and PD have representations in the form[17]

$$\mathrm{GD} = a^2 + 3by^+ \tag{5}$$

$$\mathrm{D} = -a^2 - 2by^+ \tag{6}$$

$$\mathrm{PD} = -by^+ \tag{7}$$

where

$$a^2 = (\nu^2/u_*^4)(\overline{u_{1,2}(0)^2} + \overline{u_{3,2}(0)^2})$$

and

$$b = (\nu^3/u_*^5)(\overline{u_{1,2}(0)u_{1,22}(0)} + \overline{u_{3,2}(0)u_{3,22}(0)})$$

For the data in Refs. 11 and 18, $a^2 = 0.0667$ and $b = 0.034$. The remaining terms in Eq. (1) are $\sim {y^+}^2$ or ${y^+}^3$ near $y^+ = 0$ so that, in particular, the PD + KD model represents primarily PD in this region, and Eq. (1) reduces to

$$\mathrm{GD} + \mathrm{PD} + \mathrm{D} = 0 \tag{8}$$

It is evident from Eqs. (5-7) that the sum of the negative slopes in the D and PD curves at $y^+=0$, i.e., $-2b$ and $-b$, respectively, balances that of GD, namely, $3b$. If $b=0$, as for the PD model in Eq. (2), then GD and D have zero slope at the wall. In this case, the peak in k is suppressed since the experimental data for k is only consistent with curve (a) in Fig. 4, which has a significant nonzero slope at $y^+=0$.

Figure 6 shows the complete computed energy budget for $0 \leq y^+ \leq 10$. It is evident that the calculated curves for GD and D have zero slope at the wall. If the model for PD allowed for a negative slope, then, in order to preserve the energy balance, GD would generally have to acquire a positive slope consistent with the experimental GD curve in Fig. 4. Of course it may also be possible to manipulate the ϵ equation to achieve a similar end. However, this seems unlikely in view of the apparent insensitivity of the k predictions to variations in term D as noted previously. Furthermore, this would clearly violate the proper physics of the boundary region contained in Eqs. (5-8).

It is evident that some significant improvements to the model currently used for the pressure diffusion term in the k equation are required if better predictions of peak k values near the boundaries are to be obtained using the k-ϵ closure. In addition to the constraints described herein, others of a more general kind pertaining to this correlation have been described by Speziale.[20] Considerable further investigation will be needed to develop models for the pressure-velocity correlation which satisfy all of these limitations. This will be the subject of a future study.

Acknowledgments

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